

R-Modes on Rapidly Rotating, Relativistic Stars: I. Do Type-I Bursts Excite Modes in the Neutron-Star Ocean?

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ABSTRACT

During a Type-I burst, the turbulent deflagration front may excite waves in the neutron star ocean and upper atmosphere with frequencies, $\omega \sim 1$ Hz. These waves may be observed as highly coherent flux oscillations during the burst. The frequencies of these waves changes as the upper layers of the neutron star cool which accounts for the small variation in the observed QPO frequencies. In principle several modes could be excited but the fundamental buoyant r -mode exhibits significantly larger variability for a given excitation than all of the other modes. An analysis of modes in the burning layers themselves and the underlying ocean shows that it is unlikely these modes can account for the observed burst oscillations. On the other hand, photospheric modes which reside in a cooler portion of the neutron star atmosphere may provide an excellent explanation for the observed oscillations.

Subject headings: stars : neutron, oscillations – X-rays : bursts, binaries

1. Introduction

As material accretes onto the surface of a star, the generation of nuclear energy may be stable or unstable depending on the rate of accretion and the properties of the underlying star. Unstable nuclear burning on the surface of a neutron star manifests itself as Type-I X-ray bursts (Hansen & van Horn 1975; Grindlay et al. 1976; Woosley & Taam 1976; Joss 1977; Lamb & Lamb 1978; Lewin et al. 1993; Strohmayer & Bildsten 2003, the final two are

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reviews). The accumulation of material on the surface of the star is likely to be sufficiently chaotic that the nuclear burning ignites at a particular point, so one would expect that as the burning envelopes the stellar surface that the observed flux to vary at approximately the spin frequency of the star (Joss 1978). Spitkovsky et al. (2002) and Zingale et al. (2001) present recent models for the growth of the flame over the stellar surface. The discovery of these oscillations had to wait for the launch of RXTE (Strohmayer et al. 1997). Surprisingly the frequency of the observed oscillations varied during the burst which Strohmayer et al. (1997a) argue is a hallmark of the conservation of angular momentum during the expansion and contraction of the atmosphere.

The detailed models of Cumming & Bildsten (2000) estimate the radius expansion expected during the burst. Heyl (2000) calculated the general relativistic corrections required to translate the predictions of Cumming & Bildsten (2000) into observable quantities, and found that the observed frequency shift would be 30% - 50% of that predicted by Cumming & Bildsten (2000). Although Abramowicz et al. (2001) (and later Cumming et al. 2002) found a error in the derivation by Heyl (2000), Abramowicz and colleagues also found that the general relativistic decrement is large. Cumming et al. (2002) also discovered an error in their earlier work (Cumming & Bildsten 2000) which lead them to overestimate the frequency shift by a factor of two even in the Newtonian context.

Cumming et al. (2002) pointed out that the frequency shifts observed by Wijnands et al. (2001) and Galloway et al. (2001) were too large to be accounted for by the radius expansion models of Heyl (2000) and Cumming & Bildsten (2000). This letter examines an alternate model for both the observed frequencies during Type-I bursts and their evolution (first suggested by Cumming & Bildsten 2000). The burst may excite modes in the neutron star ocean or photosphere that exhibit themselves as dark and light regions on the surface. Because these regions are associated with ocean and atmospheric waves, they move relative to the surface of the star. The properties of the atmosphere change as the burst subsides, so the natural frequency of the modes shifts accounting for the observed frequency shifts.

The following section of this letter identifies the modes excited by the burst and estimates the observational footprint of these modes. The last section examines the implications of these results.

2. Waves

Neutron stars exhibit several modes of oscillation (McDermott & Taam 1987; Lee & Strohmayer 1996). The modes that may be excited by the motion of the deflagation front

across the surface of neutron star undergoing a Type-I X-ray burst and may explain the observations of the burst oscillations share the following features.

- **They have a period on the order of one second.** The observations find that the observed frequency shift is a few cycles per second. A mode frequency (ω) of several Hertz is well matched to the timescale for the deflagration of the accumulated fuel to envelope the stars ~ 1 s (Strohmayer et al. 1997b), so it is unlikely that much slower modes would be excited. The spin frequencies of the stars (Ω) are several hundred Hertz so $q = 2\Omega/\omega \sim 10^2$.
- **The modes should travel westward.** Here, $m > 0$ denotes a westbound mode. The observed frequency is assumed to be slightly less than the spin frequency of the star, and it increases as the observed photon spectrum cools and dims; consequently, the modes should travel westward, *i.e.* in the opposite sense of the star’s rotation. Some observations have found the opposite trend during a portion of the burst, so eastbound modes may also be interesting.
- **The mode should have no latitudinal nodes,** or $l_\mu = 0$. The modes of a rotating star are squeezed near the equator. Except for special geometries, a band both above and below the equator is visible throughout the star’s rotation, so if the mode has latitudinal modes, much of the variability will be averaged out.
- **The azimuthal eigenvalue (m) should be 1 or -1.** Observations have generally found that observed oscillation has a frequency of approximately the spin frequency of the star. Miller (1999) found that 4U 1636-536 may be exceptional, so the $|m| = 2$ case will also be discussed.
- **The main observed mode should have no radial nodes in the radial displacement,** or $n = 1$ (n counts the number of radial nodes in the transverse displacement). This ensures that any modes with a similar angular dependence will be well separated in frequency, supporting the argument that only a particular mode is excited and observed.

Three prime candidates are the g –modes, the buoyant r –modes and the Kelvin modes, of the neutron star ocean and upper atmosphere. In a rotating neutron star, the modes with various values of numbers of azimuthal nodes (m) are not degenerate in frequency. The frequency of the modes depends strongly on whether the electrons contribute to the entropy of the gas. If they are degenerate, the Brunt-Väisälä frequency (N) is reduced by the ratio of the ion pressure to the total pressure; however, this reduction is mitigated by a corresponding

increase in the pressure-scale height (H). A first approximation to the frequency of a surface wave is

$$f \sim \frac{N}{2\pi} \frac{H}{R}. \quad (1)$$

I shall examine modes in a semi-degenerate material with varying contributions of ion pressure to the total pressure $\alpha \equiv P_i/P_{\text{total}}$ and non-degenerate material with varying contributions of gas and radiation pressure, $\beta \equiv P_{\text{gas}}/P_{\text{total}}$. The heat transfer is assumed to be dominated by photons scattered by electrons in the nondegenerate regime and by electrons scattered by ions in the degenerate regime.

The results of Bildsten et al. (1996) provide a touchstone for the various results. Extending their results to a semidegenerate regime yields the following estimates for the frequency of the modes in the rotating frame of the surface of the star,

$$f_{\lambda,n,D} \equiv \frac{\omega}{2\pi} = 2.37\text{Hz} \left(2\lambda \frac{T}{10^8 \text{ K}} \frac{56}{A} \frac{8 - 4\alpha - \alpha^2}{16 - \alpha^2} \right)^{1/2} \left(\frac{10 \text{ km}}{R} \right) \left\{ 1 + n^2 \left[\frac{3\pi}{2 \ln(\rho_b/\rho_t)} \right]^2 \right\}^{-1/2}. \quad (2)$$

Here A is the mean atomic weight of non-degenerate species (here, the nuclei in the ocean), ρ_b and ρ_t are the densities at the top and bottom of the excited layer (Bildsten & Cutler 1995, give the dependence on ocean depth), and $n > 0$ is the number of radial nodes. If one takes the limit as α vanishes, the Bildsten et al. (1996) result obtains, and as α approaches unity for a fixed value of A the frequency estimate actually decreases slightly (by about 40%). The frequency of the g-mode is proportional to the Brunt

The ocean lies below the burning layers from $\rho \sim 10^7 \text{ g cm}^{-3}$ to $\rho \sim 10^8 \text{ g cm}^{-3}$, yielding 4.2 for the coefficient of n^2 is Eq. 2. The frequency of the first harmonic is about half that of the fundamental. The depth of the ocean depends sensitively on the charge of the nuclei produced by the burning. The unstable burning during Type-I bursts produces mainly iron group elements (Lewin et al. 1993), so the ocean is dramatically more shallow than in the higher accretion rate Z sources where the ocean consists mainly of *CNO* elements and extends to $\rho \sim 10^{11} \text{ g cm}^{-3}$ (Bildsten & Cutler 1995).

In a nondegenerate layer, the mode frequencies are several times larger,

$$f_{\lambda,n,ND} = 14.48\text{Hz} \left(5 \frac{\lambda}{\beta} \frac{T}{10^8 \text{ K}} \frac{0.6}{\mu} \frac{4 - 3\beta}{8 - 3\beta} \right)^{1/2} \left(\frac{10 \text{ km}}{R} \right) \left\{ 1 + n^2 \left[\frac{3\pi}{2 \ln(\rho_b/\rho_t)} \right]^2 \right\}^{-1/2}. \quad (3)$$

where μ is the mean molecular weight of the material (this includes the electrons because in the non-degenerate case they contribute to both the entropy and the pressure) and β is the ratio of the gas pressure to the total pressure. To obtain these expressions, thermal bouyancy is assumed to dominate (Cumming & Bildsten 2000) and the expressions of Bildsten &

Cutler (1995) and Bildsten et al. (1996) have been scaled by the ratio of the Brunt-Väisälä frequency in the nondegenerate atmosphere (Bildsten 1998) to the degenerate ocean. If one takes $\beta \rightarrow 1$ and $\alpha \rightarrow 1$ in expressions Eq. 2 and 3 for the same composition, the two expressions agree. The increase in the frequency estimates results entirely from the decrease in the mean molecular weight of the nondegenerate species, because the electrons are no longer degenerate.

Both equations 2 and 3 include a logarithmic factor involving the density of the top and bottom of the layers. Bildsten et al. (1996) derived this factor when they considered a degenerate layer. It is unclear whether it is indeed appropriate in a non-degenerate layer. This factor changes the estimate of the mode frequencies by at most a factor of two for the situations considered here. Furthermore, in the non-degenerate regime the mode period is shorter than the thermal time for the layer, so one would expect the modes to be isothermal rather than adiabatic. Bildsten & Cutler (1995) examine this point in their appendix and find that the lack of adiabaticity changes the structure of the mode and most importantly the assumed energy in a photospheric mode. However, they did not determine how it would change the frequency of a mode restricted to an isothermal region.

Unless the temperature is small or n is large, the frequency of the oscillation will be much larger than one Hertz. The latter possibility is disfavored according to the criteria listed above. Furthermore, to meet the requirements above, $\lambda \sim 1$.

Longuet-Higgins (1968) derives the mode structure for the ocean of a rotating fluid shell which includes some modes not discussed by Bildsten et al. (1996). As q increases, for the g -modes, λ approaches

$$\lambda \sim (2\nu + 1)^2 q^2 \quad (m > 0, \nu = 0, 1, 2, \dots). \quad (4)$$

where $\nu \geq 1$ is the number of latitudinal nodes in the perturbation to the northward velocity of the fluid. $l_\mu = \nu + 1$ for g -modes.

Bildsten et al. (1996) also discuss a set of eastbound modes which Longuet-Higgins (1968) identifies as the Kelvin modes with

$$\lambda \sim m^2 \quad (m < 0, \nu = 1) \quad (5)$$

which have $l_\mu = 0$.

For the buoyant r -modes (which all travel westward), λ approaches,

$$\lambda \sim m^2 (2\nu + 1)^{-2} \quad (m > 0, \nu = 1, 2, \dots). \quad (6)$$

and $l_\mu = \nu - 1$ for the r -modes.

As $\epsilon = q^2 \lambda$ increases the modes become more and more localized near the stellar equator, and the eigenfunctions resemble parabolic cylinder functions which may be expressed as the product of a Hermite polynomial and a Gaussian. The vertical displacement (or equivalently the pressure perturbation) for the mode ζ is given by the following expressions as ϵ goes to infinity (Longuet-Higgins 1968),

$$\zeta_{g\text{-mode}} \sim \frac{1}{(2\nu + 1)^{1/2} \epsilon^{1/2}} e^{-\frac{1}{2}\eta^2} \left(\nu H_{\nu-1}(\eta) - \frac{1}{2} H_{\nu+1}(\eta) \right) e^{m\phi + \omega t} \quad (7)$$

$$\zeta_{\text{Kelvin}} \sim \frac{2}{m} \epsilon^{1/4} e^{-\frac{1}{2}\eta^2} e^{m\phi + \omega t} \quad (8)$$

$$\zeta_{r\text{-mode}} \sim \frac{2\nu + 1}{2m} \epsilon^{-1/4} e^{-\frac{1}{2}\eta^2} \left(H_{\nu-1}(\eta) + \frac{1}{2\nu + 2} H_{\nu+1}(\eta) \right) e^{m\phi + \omega t} \quad (9)$$

where $\eta = \epsilon^{1/4} \mu$ and $\mu = \cos \theta$. Most of the excitation in a mode lies between $-\sqrt{\nu} < \eta < \sqrt{\nu}$. For the different modes, this leads to the following range in μ ,

$$|\mu| < \begin{cases} q^{-1} (2 + \nu^{-1})^{-1/2} & g\text{-modes} \\ q^{-1/2} |m|^{-1/2} & \text{Kelvin modes.} \\ q^{-1/2} |m|^{-1/2} (2\nu^2 + \nu)^{1/2} & \text{buoyant } r\text{-modes} \end{cases} \quad (10)$$

The buoyant r -modes occupy the widest band near the equator; the variability is approximately proportional to the square root of the width of the band, so the r -modes should exhibit the largest variability.

The next subsection focuses on the modes which are likely to be excited in the ocean and what their observational footprint would be. The modes tend to become more localized in the equatorial regions as the spin of the star increases (Bildsten et al. 1996). The observations indicate that the excited modes typically have $|m| = 1$ (Strohmayer et al. 1997a) or possibly $|m| = 2$ (Miller 1999).

The frequency of the modes in the observer's frame is given by $\omega_I = \omega - m\Omega$ (neglecting the gravitational redshift for now), so for $m > 0$ one would see the frequency of the variation increase as the temperature of the ocean decreases. This is what is usually observed (Cumming & Bildsten 2000); therefore, the detailed calculations will focus on the $m = 1$ and $m = 2$ modes with frequencies of about one cycle per second.

2.1. Visibility

Calculating the bolometric pulsed fraction for the mode provides an estimate the visibility of the various modes. Because the frequency of the mode is about one Hertz and

the rotational frequency of the star is about 300 Hz, a value of $q = 300$ is appropriate to calculate the modes. The variability of these modes decreases as $q^{-1/4}$. The observer lies at a latitude of 30° and compare the total flux observed when the observer lies above a bright patch (intensity maximum) patch to when the observer lies above a faint patch (where the intensity vanishes). The pulsed fraction so defined is

$$\text{PF} = \frac{f(\text{bright}) - f(\text{faint})}{f(\text{bright}) + f(\text{faint})} \quad (11)$$

and can range from -1 to 1 . The pulsed fraction is negative if the total flux observed from the star is larger when the observer lies above a faint region.

Lacking a specific model for how the presense of a mode induces variations in the flux, the effective temperature of each surface element is assumed to vary as the linear combination of a constant and the vertical displacement induced by the mode of interest. The normalization of the mode is selected so that the effective temperature varies over the surface from zero to twice the underlying value. Because the observed variability is small during the tail of the burst, the particular choice of coupling is unimportant; one can always choose a different type of connection between the mode and the observed flux and rescale the size of the underlying wave.

The gravitational defocussing of the neutron star surface in the Schwarzschild geometry (*e.g.* Heyl & Hernquist 1998; Page 1995) and limb darkening also affect the variability. The limb darkening is modeled by allowing the intensity from a given surface element to vary as the cosine of the zenith angle (*e.g.* Perna et al. 2001). Figure 1 presents the results for the first few modes with $m = 1$ and $m = 2$. The inclusion of limb darkening increases the expected variability significantly and also alleviates the effects of gravitational defocussing.

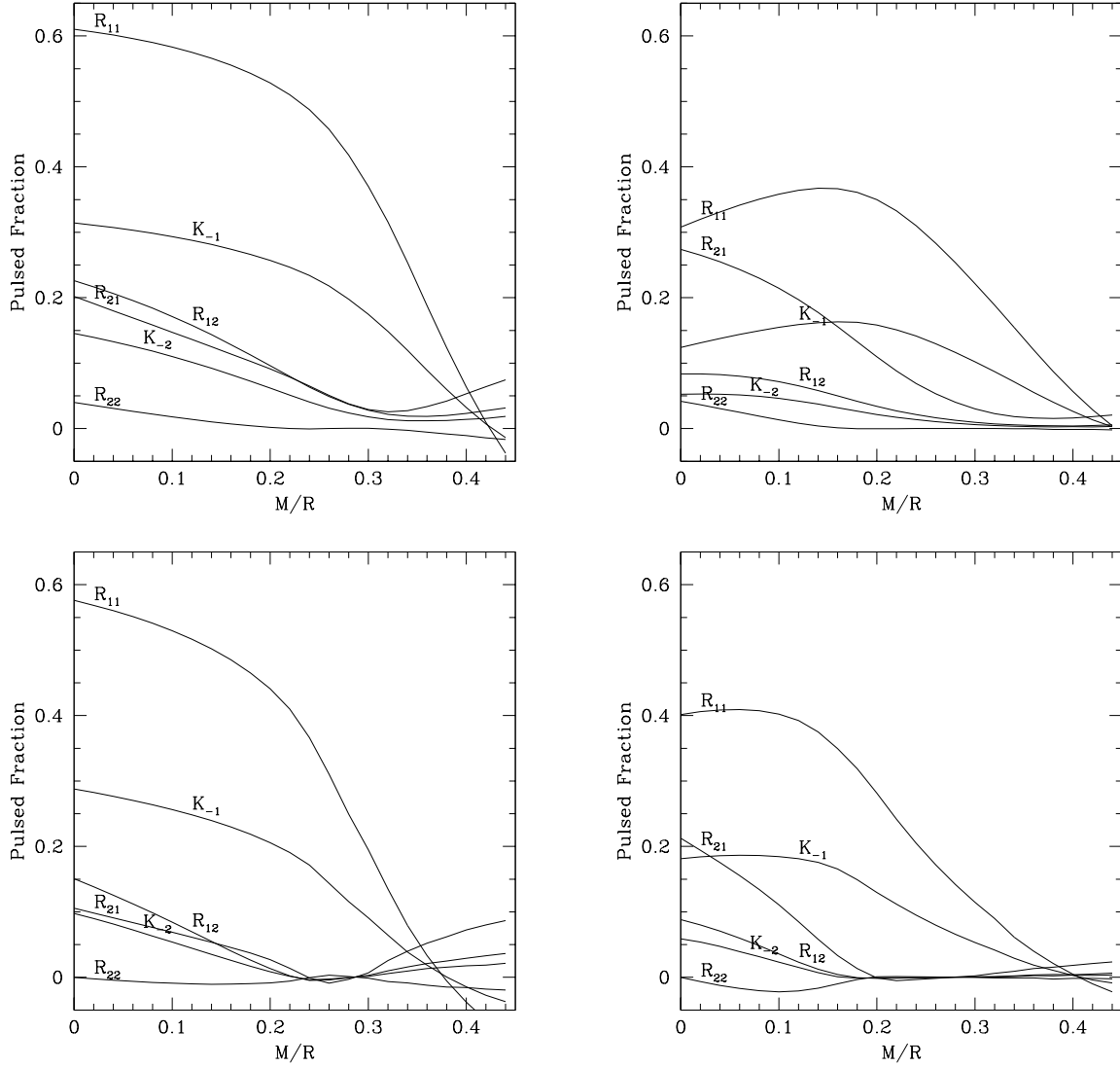


Fig. 1.— Pulsed Fraction for various modes as a function of M/R for an observer latitude of 30° (left) and 60° (right). The upper curves include a rudimentary treatment of limb darkening. A negative pulsed fraction indicates that the star is brightest when the observer is directly above a dark region. $R_{\nu m}$ denotes the Rossby mode with the particular value of ν and m . K_m denotes a Kelvin mode.

As expected from the earlier discussion, the buoyant r -modes exhibit the largest variability. Modes with a larger number of nodes ν show less variability as do modes with $|m| > 1$. The Kelvin modes are more concentrated near the equator, and therefore exhibit less variability. The g -modes which are extremely concentrated near the equator show negligible variability when averaged over the visible portion of the star with a static background. Of all the modes in the ocean for a given excitation, the lowest-order r -mode exhibits the largest variability and also satisfies the observational constraints. The absolute variability of a particular mode depends on the *local* coupling with the mode and the emergent flux; the details of this coupling are beyond the scope of this paper, but the relative variability between the different modes will not depend on these details as long as the coupling is local.

2.2. Variability Frequency

With a mode excited, a distant observer would find that the flux from the star would vary with a frequency of $m\Omega - \omega$, slightly less than the spin frequency of the star for $m = 1$.

2.2.1. A Degenerate Layer

If the mode resides in the degenerate ocean lying beneath the burning layers, the frequency of the mode is given by Eq. 15 of Bildsten et al. (1996) or Eq. 2 as $\alpha \rightarrow 0$,

$$f_{\lambda,n,D} \equiv \frac{\omega}{2\pi} = 2.37\text{Hz} \left(m^2 \frac{T}{10^8 \text{ K}} \frac{56}{A} \right)^{1/2} \left(\frac{10 \text{ km}}{R} \right) (1 + 4.2n^2)^{-1/2} \times \begin{cases} (2\nu + 1)^{-1} & r\text{-modes} \\ 1 & \text{Kelvin modes} \end{cases} \quad (12)$$

in the limit of rapid rotation, *i.e.* large $q = 2\Omega/\omega$. The ocean is left after a Type-I burst which burns the accreted hydrogen and helium directly to the iron group elements (Lewin et al. 1993) and possibly beyond (Schatz et al. 2001); for simplicity A is set at 56 ; Bildsten et al. (1996) were concerned with the carbon ocean remaining after the steady nuclear burning associated with the more rapidly accreting “Z” sources. This frequency estimate assumes that the material in the excited layer is degenerate while the mode is excited.

The temperature at the bottom of the burning layers at the peak of the burst is typically around 10^9 K and drops to 10^8 K as the burst subsides (e.g. Joss 1978). For the $\nu = 1$ and $n = 1$ westbound r -mode in the burning layer itself, the degenerate formula (Eq.12 yields a frequency shift from 1.62 Hz at the peak temperature of 2.2×10^9 K (Cumming & Bildsten 2000) to 0.35 Hz at 10^8 K. Even this modest frequency shift would require that the massive ocean be well coupled to the burning layers above. Because the mass of the ocean is much

greater than that of the burning layers, it is unlikely that the ingoing flux during the burst is sufficient to heat up the ocean to 10^9 K.

Modes with higher values of n will exhibit smaller frequency shifts — for $n = 2$ the shift is 45% smaller. While modes with $m < -1$ will have larger shifts (but the variability will be found near $|m|$ times the rotational frequency), the observed variation in the total flux is less. The Kelvin mode exhibits a frequency shift that is three times larger because λ is nine times larger (see Eq. 5). The observed frequency of the mode decreases as the ocean cools.

2.2.2. A Non-degenerate Layer

The burning layers become sufficiently hot to lift the electron degeneracy (Cumming & Bildsten 2000), the frequency shift would be an order of magnitude larger. Additionally, because I have assumed that electron scattering dominates the opacity, the ratio of the gas pressure to the total pressure (β) is constant through the non-degenerate layer where neither convection nor nuclear burning occur. If I take β equal to unity, and the temperature of the burning layer going from 10^9 K to 10^8 K while the mode is excited, the mode frequency will change by about eight Hertz much larger than is observed.

However, a mode could naturally become trapped in the photosphere itself where the gas is typically much cooler. Lapidus et al. (1994) argued that sound waves can be trapped above the photosphere of the neutron star. Modes are trapped above the photosphere of the sun in the chromosphere between the temperature minimum and the steep temperature gradient at higher altitudes (e.g. Leibacher et al. 1982). It is natural to speculate that something similar could occur in the atmosphere of a Type-I burst. Lapidus et al. (1994) considered situations where $\beta \ll 1$ and the photosphere expanded to several stellar radii (*i.e.* radius expansion bursts). Here I will focus on gravity waves in the relatively thin photospheres of burst that do not experience radius expansion.

Because β is constant through the photosphere and it is potentially observable, it is natural to use the condition of hydrostatic equilibrium to eliminate the temperature from the frequency estimate.

$$T_{\text{eff}}^4 = (1 - \beta) \frac{cg_s}{\sigma\kappa} = (1 - \beta) \frac{g_s}{2.4 \times 10^{14} \text{cm s}^{-2}} (2.4 \times 10^7 \text{K})^4 \quad \text{and} \quad 1 - \beta = \frac{F}{F_{\text{Edd}}} \quad (13)$$

This substitution in Eq. 3 yields

$$f_{\lambda,n,\text{ND}} = 7.09 \text{Hz} \left(5m^2 \frac{(1 - \beta)^{1/4}}{\beta} \frac{4 - 3\beta}{8 - 3\beta} \frac{0.6}{\mu} \right)^{1/2} (1 + 4.2n^2)^{-1/2} \times$$

$$\left(\frac{g_s}{2.4 \times 10^{14} \text{cm s}^{-2}}\right)^{1/8} \left(\frac{10 \text{ km}}{R}\right) \times \begin{cases} (2\nu + 1)^{-1} & r\text{-modes} \\ 1 & \text{Kelvin modes} \end{cases} \quad (14)$$

where I have assumed that the photosphere spans an order of magnitude in density. Two important features of Eq. 14 is that it depends on observable quantities and that the frequency drift diverges as the flux approaches the Eddington limit.

If the burst oscillation depicted in Fig. 2 of Strohmayer & Markwardt (1999) is taken as an example with the assumption that the burst peaked very close to the Eddington flux, the oscillation starts with $F/F_{\text{Edd}} \approx 0.9$ and continues until $F/F_{\text{Edd}} \approx 0.1$, which yields a frequency shift of 4.1 Hz (or 3.07 Hz including a gravitational redshift of 0.35) about twice the observed value. Strohmayer (1999) found oscillations whose frequency increased as the burst subsides. The excitation of a Kelvin mode could naturally explain this behavior. The spin-down portion of the oscillation begins when the flux is 0.6 of its peak value and continues until the fraction is 0.3 and its frequency increases by 1.4 Hz. The model predicts a slightly larger shift of 1.9 Hz (again with the gravitational redshift).

It is crucial to emphasize that the estimates of the frequency shifts given in the previous paragraph are upper bounds for a layer of a given thickness in density. If the composition of the photosphere is not solar but helium, the frequency shifts would decrease by one-third. Less subtle is the assumption that the bursts reach the Eddington rate at their peak. For example if the two example bursts discussed earlier peaked at 80% of the Eddington rate the frequency shifts would be 1.43 Hz for the spin-up example – almost exactly the value found by Strohmayer & Markwardt (1999) – and 1.30 Hz for the spin-down case. In sources with radius expansion bursts, the peak burst flux can be calibrated and these predicted trends tested in detail to possibly give hints of the radii and gravitational redshifts of the underlying neutron stars.

The observed burst oscillations have been found to have Q –values of several thousand. Since the frequency of the mode is boosted by a factor of several hundred, this is consistent with the mode having a Q –value on the order of ten. The observed value of Q is given by the product of the ratio of the observed frequency to the inherent frequency of the mode and the Q –value of the mode. Such moderate values of Q are not difficult to achieve in the neutron star ocean (Bildsten & Cutler 1995). The oscillations of interest suffer little dissipation on the timescale of the bursts. These details of the photospheric modes have not yet been determined.

3. Discussion

The preceding sections have presented a model for the observed variability in Type-I bursts and its shift in frequency. Unlike the simple rotational modulation model, the excitation and evolution of r -modes in the neutron-star ocean can naturally account for the size and sign of the shift and the presence of variability even after the burst has enveloped the entire star.

During the onset of the burst when only a portion of the star is hot, it is quite natural to account for the variability as a growing hotspot (Strohmayer et al. 1997b). Strohmayer et al. (1998) found that the pulsed fraction of $75 \pm 15\%$ during the onset of the burst. The presence and growth of this hotspot sets up travelling modes in the ocean. Due to a match in timescales between the growth of the spot and the oscillation of the modes, modes with frequencies near a Hertz are preferentially excited. Modes with $m = 1$ result in the largest observed variability. From the observer’s point of view, the flux varies at slightly less than the rotational frequency of the star. During the decay of the burst, the observed modulation is significantly lower $\sim 15\%$ (Strohmayer et al. 1998). The r -modes with the simple coupling considered in the letter can easily generate this amplitude of modulation.

Although modes with higher (more negative) values of m may be excited, the resulting variability is much smaller. Since the burst begins in a particular spot and grows, one would expect modes with odd values of m to be excited preferentially. The resulting variability for the $m = 3$ mode is a factor of 30–100 smaller than the $m = 1$ mode; this is well below the upper limits quoted by Munro et al. (2002a) for several sources.

Miller (1999) discovered that 4U 1636-536 exhibits oscillations at 580 Hz as well as much weaker oscillations 290 Hz; on the basis of other observations (QPOs) and a model of the accretion flow (Miller et al. 1998), Miller argues that the latter is the spin frequency of the star and the former is its first overtone. He associates the 580 Hz signal with the presence of two nearly antipodal hotspots burning on the surface (Weinberg et al. 2001). Recent observations of burst oscillations from SAX J1808.4-3658 at the known spin frequency of about 400 Hz clarifies this issue (Chakrabarty et al. 2003). The difference in the frequencies of the QPOs is only about 200 Hz, so in other sources where the burst oscillation frequency is twice the QPO difference, the burst frequency may also be the spin frequency of the star, so the $|m| = 1$ mode is indeed the correct one to consider.

On the other hand, if 290 Hz is indeed the spin frequency of 4U 1636-536, in these particular bursts, anomalously, the $m = 2$ mode is excited more strongly than the $m = 1$ mode. If both modes have the same value of n (the number of radial nodes), a prediction of the model would be that the frequency shift would be twice as large for the 580 Hz variation

as for the 290 Hz variation. The simple rotational modulation model predicts a similar trend. However, the value of ω could be chosen by the evolution of the burning front, so the two modes could correspond to different values of n with the nearly same value of ω . In this case, both oscillations would vary in frequency by the same amount.

This brings up the important question of why the burst only excites a mode with a particular number of radial nodes. If this were not the case, power would appear at several frequencies near the spin frequency of the star and these frequencies would all evolve according to Equation (3). Perhaps it is not surprising that only a mode with a $n = 1$ is excited since the excitation of higher modes requires additional shearing in the middle of the excited layer. The burst is associated with activity only in a thin layer at the top of ocean or at the bottom of the photosphere.

This model makes several predictions. In principle eastbound modes may be excited. This would result in an observed frequency decrease as the atmosphere cools after the burst about three times larger than that caused by the westbound modes (e.g. Strohmayer 1999). More generally, several modes could be excited. If several modes were excited, one could in principle derive properties of the neutron star since the spacing of the modes is well understood. These higher order modes would typically exhibit less variability since the variability is proportional to $q^{-1/4}$ or $\omega^{1/4}$.

Furthermore, since the frequency of the mode changes by a factor of several as the surface layers cool, one would expect the variability to decrease slightly during this epoch as well. Additionally, more quickly rotating stars should exhibit slightly less variability and more importantly similar frequency shifts, *i.e.* the frequency shift is not proportional to Ω in contrast with standard, angular-momentum conservation model. Cumming & Bildsten (2000) noted that the frequency shift rather than the fractional frequency shift is similar from source to source. Also, because the mode may still be excited after the ocean cools to its equilibrium temperature, one need not expect the observed frequency of the oscillation to asymptotically approach the precisely same value from one burst to the next.

Muno et al. (2002b) noted that the asymptotic frequency is stable to a few parts per thousand from burst to burst. For the photospheric model, the limit of the observed frequency as the burst flux goes to zero is indeed the spin rate of the underlying neutron star and should be constant from burst to burst. Muno et al. (2002b) also found that a small fraction of bursts exhibited multiple oscillation frequencies or spin-down episodes which this model exhibits. Most interestingly they found that the burst oscillations cease during radius expansion episodes. During a radius expansion episode the frequency of the photospheric modes diverges so the photospheric model argues that no oscillations should be found during radius expansion.

Cumming & Bildsten (2000) argued that even a small magnetic field could play a dynamic role in the frequency shift of burst oscillations. In their layer model, the shearing fluid could dramatically stretch the magnetic-field lines which could provide a significant resistance to the shearing motion and limit the expected frequency shifts. In this model, the fluid itself does not move a significant distance relative to the stellar surface so the magnetic field does not get stretched significantly. In principle a sufficiently strong magnetic field could change the mode frequencies (e.g. Morsink & Rezanian 2002, see Eq. 1) but a detailed discussion of the effects of weak magnetic fields on these models is beyond the scope of these article.

This letter proposes a model for Type-I burst oscillations in which the burst excites waves in either the degenerate ocean, the burning layer itself or the photosphere. The frequency in the rotating frame of the waves in the ocean and the photosphere are typically several Hertz which provides a good match with the deflagration timescale. The frequency in the burning layers is a factor of ten larger.

Even though the ocean model does yield mode frequencies near the observed values, its thermal inertia is large so it is difficult to understand how its properties could change so dramatically as the burst subsides. The photospheric model best accounts for the observed frequency shifts. The predicted frequency shift depends on the potentially observable value of the ratio of the outgoing flux to the Eddington value, and it is close to the observed values.

However, further work is necessary. The frequency estimates presented here were derived using several simplify assumptions. First, it was assumed that the mode frequencies scale simply with the Brunt-Väisälä frequency as one goes from a degenerate to a non-degenerate to a radiation-pressure-dominated regime. Second, the adiabatic estimates of Bildsten & Cutler (1995) and Bildsten et al. (1996) were used in the photosphere where a more realistic isothermal approximation would be appropriate. Third, the magnetic field of the neutron star could also affect the frequency of modes in the tenuous photosphere (see Bildsten & Cutler 1995, for a discussion). A more detailed treatment of the physics of the photosphere of a neutron star during a Type-I burst could address all of these concerns as well as determine the photospheric structure of a neutron star during a Type-I burst and would be very worthwhile given the approximate agreement between these gravity-wave models and the observations.

In general these gravity-wave models present the possibility of a family of modes being excited during the burst whose observed frequencies may increase or decrease as the ocean cools. Modes with more radial, azimuthal and latitudinal nodes tend to exhibit less variability when averaged over the observed portion of the stellar surface – the mode which lacks latitudinal and radial nodes and has only one azimuthal mode exhibits a significantly larger variability than the other modes, independent of viewing angle. Finally, the magnitude of

the frequency shift should be independent of the spin rate of the star, in contrast to the standard picture of Type-I burst oscillations.

Support for this work was provided by the National Aeronautics and Space Administration through Chandra Postdoctoral Fellowship Award Number PF0-10015 issued by the Chandra X-ray Observatory Center, which is operated by the Smithsonian Astrophysical Observatory for and on behalf of NASA under contract NAS8-39073. I would like to acknowledge useful discussions with Greg Ushomirsky, Avi Loeb and Dimitrios Psaltis and helpful comments from the referee.

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